8.1 - Defining and Using Sequences and Series

Sequence - an ordered list of numbers
Term - one number in the list

Arithmetic Sequence 3, 8, 13, 18, ?? 23

1 2 3 4 5

Geometric Sequence 2, 6, 18, 54, ?? 162

Infinite Sequence 2, 6, 18, 54, . . .

8.1 - Defining and Using Sequences and Series

Arithmetic Sequence

Find the common difference and then find the next three terms.

$d = ?$ $\quad 21, 27, 33, ??$

$d = 6$ $\quad 21, 27, 33, 39, 45, 51$

Practice

1. $21, 15, 9, \ldots$

$d = -6$

$3, -3, -9$

2. $9.9, 13.7, 17.5, \ldots$

$d = 3.8$

$21.3, 25.1, 28.9$

8.1 - Defining and Using Sequences and Series

Arithmetic Sequence

Find the common difference

$d = ?$ $\quad \frac{1}{3}, \frac{\sqrt{3}}{3}, \ldots \quad$ $d = \frac{\sqrt{3} - 1}{3}$

Rule for Arithmetic Sequence

$$a_n = a_1 + (n-1)d$$

8.1 - Defining and Using Sequences and Series

Arithmetic Sequence

Find the common difference and then find the next three terms.

$d = ?$ $\quad 21, 27, 33, ??$

$d = 6$ $\quad 21, 27, 33, 39, 45, 51$

Find the 10th term given d and the first term.

$d = 4$

$a_1 = 3$

$a_n = a_1 + (n-1)d$

$a_{10} = 3 + (10-1)4$

$a_{10} = 39$

Practice

1. 15th term

$d = -4$

$a_1 = 65$

$a_{15} = 9$

2. 19th term

$d = 2.5$

$a_1 = -22$

$a_{19} = 23$

8.1 - Defining and Using Sequences and Series

Arithmetic Sequence

Find the 10th term given d and the first term.

$d = 4$

$a_1 = 3$

$a_n = a_1 + (n-1)d$

$a_{10} = 3 + (10-1)4$

$a_{10} = 39$

Given the following, find the first term.

$a_2 = 8$

$a_4 = 12$

$d = \dfrac{12-8}{4-2} = 2$

$a_n = a_1 + (n-1)d$

$12 = a_1 + (4-1)2$

$a_1 = 6$

Practice

1. Find a_1

$a_4 = 28$

$a_{10} = 46$

$a_1 = 19$

2. Find a_3

$a_5 = -17$

$a_{13} = -81$

$a_1 = 15$

$a_3 = -1$

8.1 - Defining and Using Sequences and Series

Arithmetic Sequence

Given the sequence, find the rule.

$0, 3, 6, 9, \ldots$

$$a_n = a_1 + (n-1)d$$

$$a_n = 0 + (n-1)3$$

$$a_n = 3n - 3$$

Practice - Find the rule for a_n

1. $4, 7, 10, 13, \ldots$

$a_n = 3n + 1$

2. $10, 5, 0, -5, \ldots$

$a_n = -5n + 15$

8.1 - Defining and Using Sequences and Series

Arithmetic Series

Sigma Notation

$$\sum_{n=1}^{3} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} \qquad \sum_{n=1}^{3} \frac{n}{n+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4}$$

$$\sum_{n=1}^{\infty} \frac{n}{n+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \ldots$$

Practice - Find the sigma notation for the series

1. $\frac{1}{5} + \frac{2}{4} + \frac{3}{3} + \frac{4}{2}$

$$\sum_{n=1}^{4} \frac{n}{6-n}$$

2. $\frac{\sqrt{3}}{2} - \frac{3}{4} + \frac{3\sqrt{3}}{8} - \frac{9}{16}$

$$\sum_{n=1}^{4} (-1)^{n-1} \left(\frac{\sqrt{3}}{2} \right)^{n}$$

8.1 - Defining and Using Sequences and Series

Arithmetic Sequence

3, 8, 13, 18, ??

3, 8, 13, 18, 23

Geometric Sequence

2, 6, 18, 54, ??

2, 6, 18, 54, 162

8.1 - Defining and Using Sequences and Series

Geometric Sequence

Find the common ratio and then find the next term.

$r = ?$ $\quad 2, 6, 18, 54, 162, ?$ $\quad a_n = a_1 \cdot r^{n-1}$

$r = 3$ $\quad 2, 2 \cdot 3, 2 \cdot 3^2, 2 \cdot 3^3, 2 \cdot 3^4, \ldots$ $\quad a_6 = 2 \cdot 3^{6-1}$

$a_6 = 486$

Practice

1. $20, 30, 45, \ldots$

Find a_6

$r = 1.5$

$a_6 = 151.875$

2. $90, 30, 10, \ldots$

Find a_5

$r = \frac{1}{3}$ $\quad a_5 = \frac{10}{9}$

8.1 - Defining and Using Sequences and Series

Geometric Sequence

Find the missing terms

81, ___, ___, 3

$$a_4 = a_1 \cdot r^{4-1}$$

$$3 = 81 \cdot r^3$$

$$r = \frac{1}{3}$$

$$a_2 = 81 \cdot \left(\frac{1}{3}\right)^{2-1} = 27$$

$$a_3 = 81 \cdot \left(\frac{1}{3}\right)^{3-1} = 9$$

Practice

1. 3, ___, ___, ___, 48

3, 6, 12, 24, 48

3, -6, 12, -24, 48

8.1 - Defining and Using Sequences and Series

Geometric Sequence

Find the geometric rule

$a_1 = 4$

$a_3 = 8$

$a_7 = ?$

$$\frac{a_3}{a_1} = \frac{8}{4} = r^{3-1}$$

$$2 = r^2$$

$$r = \pm\sqrt{2}$$

$$a_n = 4 \cdot (-\sqrt{2})^{(n-1)}$$

$$a_7 = 32$$

$a_1 = ?$

$a_4 = 1 + \sqrt{2}$

$a_6 = 2 + 2\sqrt{2}$

$$a_1 = \pm\frac{\sqrt{2}+2}{4}$$

8.1 - Defining and Using Sequences and Series

Geometric Sequence

Find the geometric rule

$1000, 200, 40, 8, \ldots$

$$a_n = 1000 \cdot \left(\frac{1}{5}\right)^{n-1}$$

Practice

1. $3, -6, 12, -24, \ldots$

$$a_n = 3(-2)^{n-1}$$

8.1 - Defining and Using Sequences and Series

Geometric Sequence

Find a_7

$a_2 = 20$

$a_5 = 2500$

$$\frac{a_5}{a_2} = \frac{a_1 r^4}{a_1 r^1} = r^3 = \frac{2500}{20} = 125$$

$r = 5$ $\quad a_2 = 20 = a_1(5)^1 \quad a_1 = 4$

$a_7 = 4(5)^6 = 62{,}500$

Practice

1. $a_2 = 1;\ a_5 = 27$

Find a_8

$a_8 = 729$

2. ___, ___, -12, ___, ___, 96

-3, 6, -12, 24, -48, 96